Distance and Midpoint Formulas

Reporting Category	Reasoning, Lines, and Transformations	
Topic	Developing and applying distance and midpoint formulas	
Primary SOL	G.3a	The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include investigating and using formulas for finding distance, midpoint, and slope.
Related SOL	G.3b, G.3c, G.3d, G.8, G.12	

Materials

- Activity Sheets 1, 2, and 3 (attached)

Vocabulary

right triangle, hypotenuse, leg (of a right triangle) distance, length, x*-coordinate,* y*-coordinate, ordered pair, square, square root, absolute value, Pythagorean Theorem, midpoint, average,* (earlier grades)

Student/Teacher Actions (what students and teachers should be doing to facilitate learning)

1. Have students work in small groups to complete Activity Sheet 1. Each student should record his/her own findings. Have students discuss findings with their group, and then discuss findings as a whole class.
2. Have students work in small groups to complete Activity Sheet 2. Each student should record his/her own findings. Have students discuss findings with their group, and then discuss findings as a whole class.
3. Have students work in small groups to complete Activity Sheet 3. Each student should record his/her own findings. Have students discuss findings with their group, and then discuss findings as a whole class.

Assessment

- **Questions**
  - Find a point Q that is 10 away from the point $P(8, -1)$ and has x-coordinate 2. Explain your reasoning.
  - $M(1, -2)$ is the midpoint of $\overline{AB}$, and A has coordinates (3, 4). Find the coordinates of B. Explain your reasoning.
- **Journal/Writing Prompts**
  - Summarize the activity in your journal.
  - Write a practical problem and solution that uses the distance formula (or midpoint formula.)
  - Explain whether distance can ever be negative. Justify your answer.
- **Other**
  - Have groups present their findings to the class.

  - Have groups of four students construct a short quiz covering the information presented in the class in this lesson and administer it to another group in the class.
  - Have students find the distance between two other islands on Lake Geometria.
  - Have students find a point (or a point on land) that is the same distance from two other islands.

Extensions and Connections (for all students)

- Have students estimate the area of Lake Geometria or Euclid.
- Ask students to find a point that is the same distance from three islands (circumcenter).

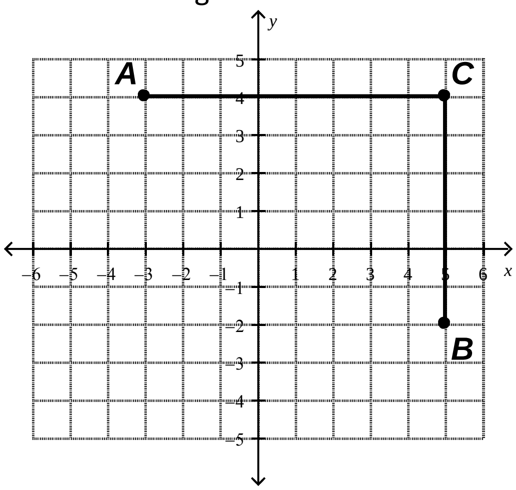
Strategies for Differentiation

- Depending on the level of students, this activity can be done independently by students or as a teacher-led activity. Use of an interactive whiteboard is encouraged.
- Give students the formulas, and have them go through the activity, and see whether they get the same formulas.
- More space and larger graphics may be necessary.
- Use a grid on the floor.

Activity Sheet 1: Deriving the Distance Formula

Name ________________________________ **Date** ____________________

1. Use the diagram below to answer the following questions.



   a. What is *AC*?

   b. What are the coordinates of *A* and *C*?

   c. Use the coordinates of *A* and *C* to compute *AC*. Show your work.

   d. What are the coordinates of *C* and *B*?

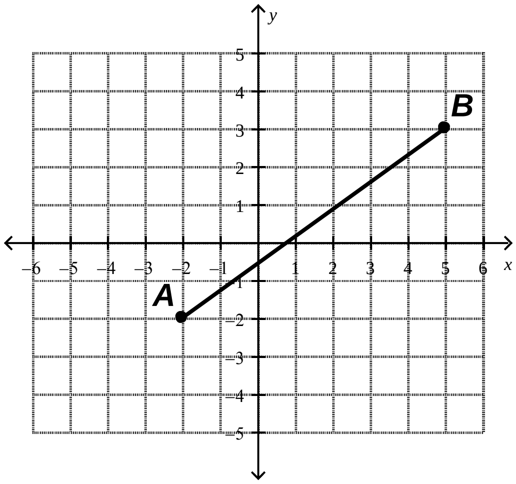
   e. Use the coordinates of *C* and *B* to compute *CB*. Show your work.

   f. Draw the segment $\overline{AB}$. What kind of triangle is ΔABC? For that kind of triangle, what are $\overline{AC}$ and $\overline{CB}$ called? What is $\overline{AB}$ called?

   g. What theorem can you use to find the length of the hypotenuse of a right triangle if you know the lengths of the two legs?

   h. Use your answer to g (above) to find *AB*.

2. Use the diagram below to answer the following questions.

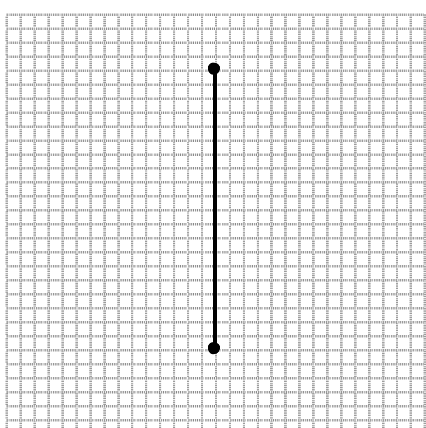


   a. On the diagram above, create a right triangle with a horizontal leg, vertical leg, and hypotenuse $\overline{AB}$. Label the vertex of the right triangle *C*. Is this the only right triangle you could have drawn?

   b. Find *AC* and *CB*.

   c. Use *AC* and *CB* to find *AB*.

3. The endpoints of a vertical segment $\overline{AB}$ are $A(x_1, y_1)$ and $B(x_2, y_2)$. Use this diagram for the following questions. *(Do not count. Graph is not to scale.)*



   a. Label the lower point $A(x_1, y_1)$ and the upper point $B(x_2, y_2)$**.** Since $\overline{AB}$ is a vertical segment, what can you say about x_1 and x_2?

   b. Express AB in terms of y_1 and y_2.

   c. Express AB in a different way in terms of y_1 and y_2.

d. Why is it necessary to use absolute value for the formulas above?

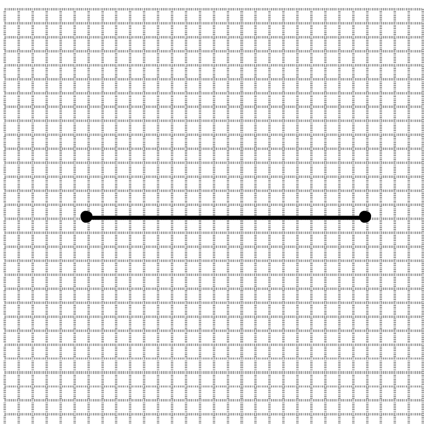
e. Does it matter which of the two formulas above you use?

f. Write a formula for *AB* using either formula above.

AB =

g. The endpoints of a vertical segment are *G*(−10, 12) and *H*(−10, −22). Use one of your formulas to compute *GH*.

4. The endpoints of a horizontal segment $\overline{CD}$ are $C(x_1, y_1)$ and $D(x_2, y_2)$. Use this diagram for the following questions. *(Do not count. Graph is not to scale.)*



a. Label the point on the left $C(x_1, y_1)$ and the point on the right $D(x_2, y_2)$**.** Since $\overline{CD}$ is a horizontal segment, what can you say about y_1 and y_2?

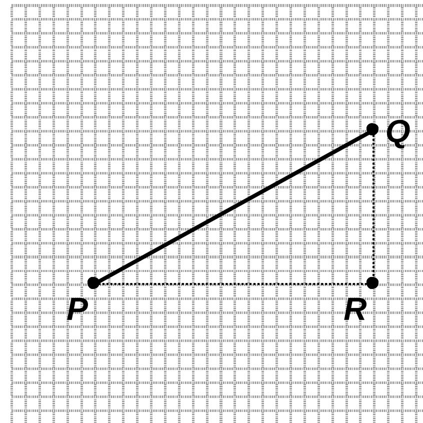
b. Express *CD* in two different ways in terms of x_1 and x_2.

c. Write a formula for *CD* using either formula above.

CD =

d. The endpoints of a horizontal segment are *E*(−10, 12) and *F*(24, 12). Use your formula to compute *EF*.

5. The endpoints of a segment $\overline{PQ}$ are $P(x_1, y_1)$ and $Q(x_2, y_2)$. Use this diagram for the following questions. (Do not count. Graph is not to scale.)



   a. Use the Pythagorean Theorem on the right triangle, using $\overline{PQ}$, $\overline{PR}$, and $\overline{QR}$. Complete the equation below:

      $(PQ)^2$ =

   b. Label the point *P* as (x_1, y_1) and point *Q* as (x_2, y_2). Find the coordinates of the point *R*.

   c. Write formulas for *PR* and *QR* using either formula above.

      PR =   *QR* =

   d. Determine whether this equation is true. If it is true, explain why. If it is false, give a counterexample. (Hint: When you square a real number, is it ever negative?)

      $|a|^2 = a^2$

   e. Use the last three problems to get a formula for $(PQ)^2$ in terms of x_1, x_2, y_1, and y_2.

      $(PQ)^2$ =

   f. Take the square root of both sides of your last formula to write *PQ* in terms of x_1, x_2, y_1, and y_2.

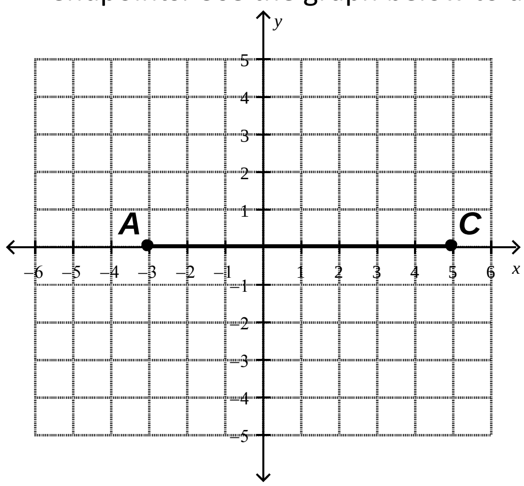
      PQ =

   g. The endpoints of a segment are *I*(8, 12) and *J*(2, 4). Use the formula you found to compute the distance between *I* and *J*.

Activity Sheet 2: Deriving the Midpoint Formula

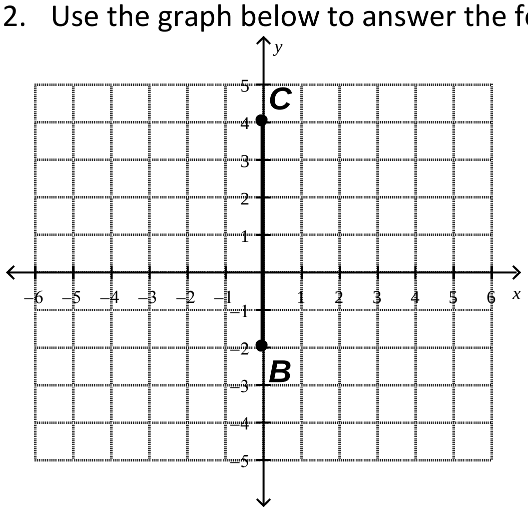
Name ________________________________ **Date** ____________________

1. The *midpoint* of a segment is the point on the segment that is the same distance from both endpoints. Use the graph below to answer the following questions.



   a. What are the *x*-coordinates of *A* and *C*? ________ and _________
   b. What is the average of the *x*-coordinates of A and C? ________
   c. What number is halfway between the *x*-coordinates of A and C? ________
   d. What are the coordinates of the midpoint of $\overline{AC}$? ________ Graph and label the midpoint.
   e. Explain the relationships among the answers to questions b, c, and d.

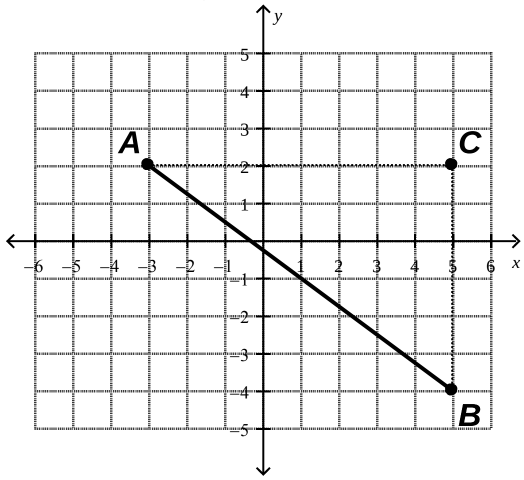
2. Use the graph below to answer the following questions.



   a. What are the *y*-coordinates of *B* and *C*? ________ and _________
   b. What is the average of the *y*-coordinates of B and C? ________
   c. What number is halfway between the *y*-coordinates of B and C? ________
   d. What are the coordinates of the midpoint of $\overline{BC}$? ________ Graph and label the midpoint.

e. Explain the relationships among the answers to questions b, c, and d.

3. Use the diagram below to answer the following questions.

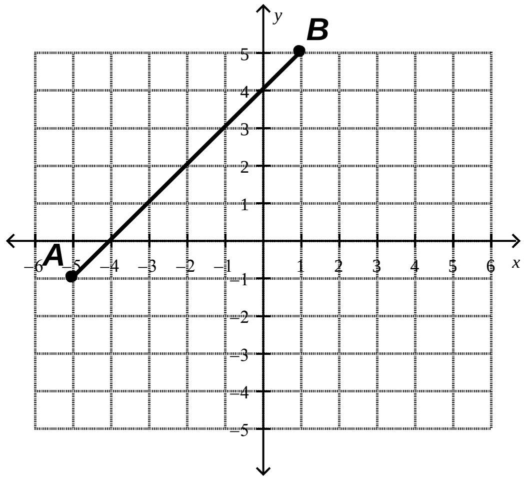


a. Graph the midpoint of $\overline{AC}$. Label it *P*. What is the *x*-coordinate of this point? ______
b. What is the average of the *x*-coordinates of A and C? ______ How is this related to your answer to a?

c. Graph the midpoint of $\overline{BC}$. Label it Q. What is the y-coordinate of this point? ______
d. What is the average of the *y*-coordinates of B and C? ______How is this related to your answer to c?

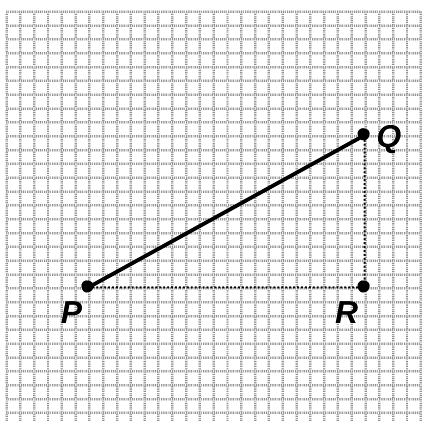
e. What is the average of the *x*-coordinates of A and B? ______
f. What is the average of the *y*-coordinates of A and B? ______
g. What is the midpoint of $\overline{BC}$?
h. How is the midpoint of $\overline{AB}$ related to the answers to 3e and 3f?

4. Use the diagram below to answer the following questions.



   a. What is the average of the *x*-coordinates of *A* and *B*? _______
   b. What is the average of the *y*-coordinates of *A* and *B*? _______
   c. What are the coordinates of the midpoint of $\overline{AB}$? _____________
   d. Explain the relationships among the answers to questions b, c, and d.

5. The endpoints of a segment $\overline{PQ}$ are $P(x_1, y_1)$ and $Q(x_2, y_2)$.



   a. Label the point *P* as (x_1, y_1) and point *Q* as (x_2, y_2).
   b. Write a formula for the average of the *x*-coordinates of *A* and *B*.

   c. Write a formula for the average of the *y*-coordinates of *A* and *B*.

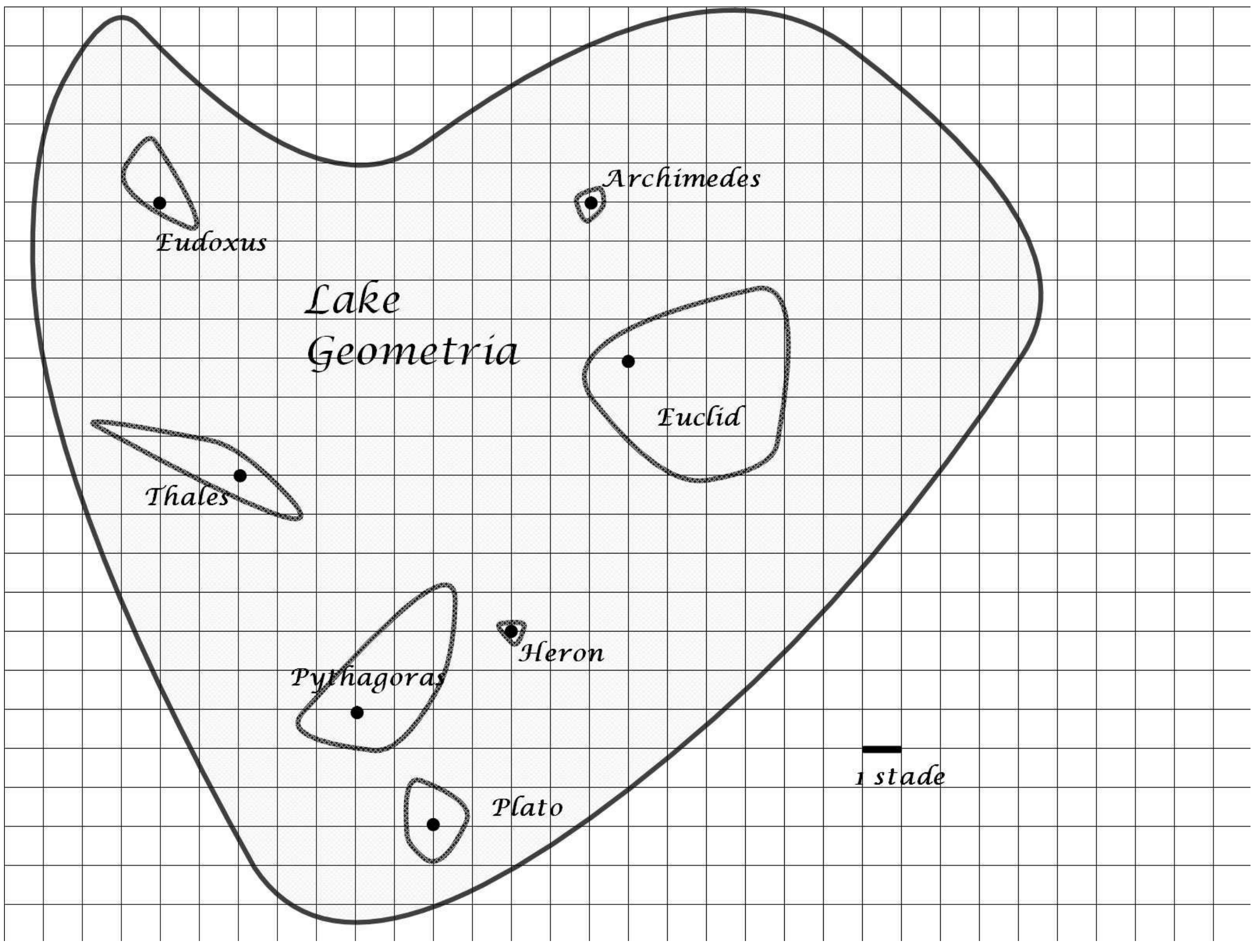
   d. One way to think of the midpoint of $\overline{PQ}$ is as follows: average of the *x*-coordinates, average of the *y*-coordinates. Use this to derive a formula for the midpoint of $\overline{PQ}$.

   e. The endpoints of a segment are *E*(8, 12) and *F*(2, 4). Use your formula to compute the midpoint of $\overline{EF}$.

Activity Sheet 3: Lake Geometria

Name ________________________________ **Date** ____________________

The islands of Lake Geometria are shown below. A cabin is marked on each island. The scale, using units called stades, is shown in the lower right. A stade measures about 600 feet, so Lake Geometria is not very big. Use the grid to help you answer the following questions. All island measurements should be made from cabin to cabin.



1. What is the distance in stades from Eudoxus to Archimedes? (Round to the nearest stade.) Describe how you found your answer.

2. What is the distance in stades from Thales to Euclid? (Round to the nearest stade.) Describe how you found your answer.

3. Which is closer to Thales—Pythagoras or Heron? Describe how you found your answer.

4. Find a point **in the water** that is the same distance from Archimedes and Eudoxus. Label the point *M*. Describe how you found this point.

5. Now find a point **on land** that is the same distance from Archimedes and Eudoxus. Label this point *N*.

6. How many points can you find that are the same distance from Archimedes and Eudoxus? Explain.

7. Find a point **on land** that is the same distance from Thales and Pythagoras. Is your point on the mainland or on an island? Does it have to be?

8. Groups staying on any of the islands are provided with solar-charged walkie-talkies, but their ranges are only about 1 mile. There are about 9 stades in a mile. With which islands could someone staying at the cabin on Thales expect to be able to communicate? Show your work, or explain how you found your answer.

9. Estimate the shortest distance from Thales to the mainland. (You may use stades, feet, or miles.) Show your work or explain how you found your answer.

10. Estimate how many miles wide Lake Geometria is at its widest point. Explain how you found your answer.